systems of ordinary differential equations

(linear differential equations or linear algebraic equations)

**Degenerate systems**

|  |  |
| --- | --- |
| **Infinitely Many Solutions** | The system has more than one solution. This typically occurs when the system is underdetermined, meaning there are fewer independent equations than unknowns. |
| **No Solutions** | The system has no solution at all. This usually happens when the system is inconsistent, meaning that the equations contradict each other. |

Consistency and rank of the coefficient matrix and augmented matrix of the system

**Check for:**

**Consistency**

(i.e., if there exists at least one solution).

* If the system is inconsistent (the equations contradict each other), it has no solution.
* If the system is consistent, proceed to the next step.

**Check for Uniqueness**

(if the system has a unique solution or infinitely many solutions).

* If the number of linearly independent equations equals the number of unknowns, and the system is consistent, it has a unique solution.
* If the number of linearly independent equations is less than the number of unknowns, and the system is consistent, it has infinitely many solutions.

[Question 1]

*ASS1*

*Determine whether the system*

*is degenerate. In the degenerate case, decide whether it has no solution or in definitely many solutions. If it has no solution, explain why, else find the general form of the solutions.*

The accent symbols on and ​ represent:

denotes the time derivative of

denotes the time derivative of

the given system of differential equations can be rewritten as:

[1] Rewrite the given system of differential equations

[2] Combine the Equations

We have:

&

[3] substitute x into [2]

Therefore, equation (1) is consistent with the substitution.

[4] substitute x into [2]

Therefore, equation (1) is **inconsistent** with the substitution.

Therefore, the system is **inconsistent &** no values of 𝑥(𝑡) and *y*(*t*) that can simultaneously satisfy both differential equations.

**Triangular systems**

Systems of equations (usually linear or differential equations) that are structured in such a way that one equation involves only one unknown, the next equation involves that unknown and one additional unknown, and so on

This structure allows for a step-by-step solution process, typically starting from the simplest equation and then using its solution to solve the next equation, and so on.

Where , ... are differential operators, and

are known functions.

e.g. in *the system*

We can transform this into triangular form by solving for one variable in terms of the other and then substituting back.

The goal is to isolate the highest derivative of one variable in the first equation and then use that to solve for the other variable in the second equation.

*is equivalent to both the following triangular systems:*

first-order equation involves both and

second-order differential equation involving only

*and*

This equation can then be used to solve for *x* once *y* is known

second-order differential equation involving only

**Solving a Triangular System:**

For the first system, you would:

1. Solve for 𝑥.
2. Use that solution to solve for 𝑦.

For the second system, you would:

1. Solve for 𝑦.
2. Use that solution to solve for 𝑥.

[Question 2]

*Show that the system:*

*is equivalent to both the following triangular systems:*

second-order differential equation involving only

first-order equation involves both and

*and*

This equation can then be used to solve for *x* once *y* is known

second-order differential equation involving only

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Type** | **Explanation** |
|  | Function | dependent variables in the system of differential equations |
|  | Function | dependent variables in the system of differential equations |
|  | Function | independent variables in the system of differential equations  interpreted as time.  depends on t  depends on t |
|  | Function | base of the natural logarithm  represents an exponential function of |
|  | Differential Operator | represents the differential operator with respect to |

* "Function" indicates that the symbol represents a variable or expression that depends on 𝑡*t*.
* "Differential Operator" indicates that the symbol represents an operator that acts on functions, differentiating them with respect to 𝑡*t*.

[1] Rewrite the given system of differential equations

(operator notation)

[2] Solve for y

[3] Solve for x

*First triangular system:*

[4] substitute y into [1]

Which matches in the first triangular system:

[5] substitute y into [2]

(this is redundant, but is just another approach to get to the same equation)

Which matches in the first triangular system:

[6]

Which matches in the first triangular system:

~~[4] substitute x into [2]~~